A New Model of Holographic QCD and Chiral Condensate in Dense Matter

Shigenori Seki[†]

Center for Quantum Spacetime (CQUeST)
Sogang University, Sinsu-dong, Mapo-gu, Seoul 121-741, Republic of Korea

Sang-Jin Sin[‡]

Department of Physics, Hanyang University, Seoul 133-791, Republic of Korea

Abstract

We consider the model of holographic QCD with asymptotic freedom and gluon condensation in its vacuum. It consists of the color D4-branes and D0-branes as a background and the flavor D8-branes as a probe. By taking a specific field theory limit, the effective coupling decreases to vanish in UV region. We then introduce the uniformly distributed baryons in terms of the baryon vertices and study the density dependence of chiral condensate, which is evaluated using the worldsheet instanton method. In the confined phase, the chiral condensate as a function of density monotonically decreases in high baryon density. Such behavior is in agreement with the expectation, while in extremely low density it increases. We attribute this anomaly to the incorrect approximation of uniformity in very low density. In the deconfined phase the chiral condensate monotonically decreases in the whole region of density.

26 April 2013

[†] sigenori@sogang.ac.kr
[‡] sjsin@hanyang.ac.kr

1. Introduction

Chiral symmetry is one of the most important properties which control the behavior of the strongly interacting nucleon system and therefore studying its order parameter, the chiral condensate, especially in the dense matter is the most urgent and important to understand the properties of hadrons in nuclei, neutron stars and so on. So far many speculations have been made about the behavior of the chiral condensate in dense medium without definite proof. One of the intuitive and acceptable ones is that the chiral condensate has a finite value at zero density and vanishes at a certain high density where the chiral symmetry is restored, and that it interpolates between the two by monotonically decreasing behavior [1]. However, no one succeeded in proving this from a first principle in a theory with strong coupling since there has not been any reliable calculational tools for the strongly interacting system especially in the presence of the dense fermions.

Since the gauge/gravity correspondence [2] is good for strongly interacting gauge theory, it is tantalizing to see what happen if we apply this method to QCD and such application is called holographic QCD. The flavor physics of QCD has been studied in many models of holographic QCD, for example, the D3/D7 model [3], the D4/D6 model [4] and the D4/D8 model [5,6]. One can incorporate baryons into those models by the use of the baryon vertices [7], the D5-branes wrapping S^5 in the D3-branes background or the D4-branes wrapping S^4 in the D4-branes background. Such models with baryonic medium have been attracting many interests [8–15]. However we have not found a model which has the behavior of chiral condensate in complete agreement with the speculation mentioned above.

Although the D4/D8 model is one of the most successful models of holographic QCD, it has several shortcomings: current quark mass can not be incorporated, and as a consequence the chiral condensate can not be naturally introduced. Various prescriptions for this problem have been suggested by [16–22]. The method by [19] allows us to calculate the quark mass and chiral condensate $\langle\bar{\psi}\psi\rangle$ from the open Wilson line, whose expectation value is given by the minimal surface of the open string worldsheet surrounded by the D8-branes, the worldsheet instanton. Therefore the chiral condensate can be evaluated by the area of that minimal surface in the analogy of the computation of Wilson loop by [23]. Following this method, we have studied the chiral condensate in both confined and deconfined phases [15]. There, we found that the chiral condensate decreases in low density but increases in high density so that the low density behavior in this model agrees with the expectation, but the high density one does not, as in all other known models.

The D4/D6 and D4/D8 models use the common background yielded by the D4-branes, in which the gluon sector of four-dimensional dual gauge theory becomes non-supersymmetric due to an anti-periodic S^1 compactification of fermions [24]. This is an advantage of this background, because the ordinary QCD does not have supersymmetry. However the effective coupling e^ϕ diverges as one goes to UV, namely, the theory has opposite behavior to asymptotic freedom. In this paper we improve this problem by introducing D0-branes smeared on the D4-branes. We take a specific field theory limit to achieve the asymptotic freedom. In general, the Dp-branes smeared on the D$(p+4)$-branes play the role of instantons, so that the Dp-branes provide a gluon condensate to the field theory on D$(p+4)$-branes [25].

The rest of the paper goes as follows. In Section 2 we suggest the new model whose background is given by taking the specific field theory limit of the supergravity solution of the D4-branes on which the D0-branes are smeared. In Section 3 we consider the D8-branes in the background of confined phase and then numerically calculate the chiral condensate depending on the density of baryons introduced as baryonic D4-branes (baryon vertices). In Section 4 we comment on the chiral condensate in deconfined phase which is analyzed in the similar way to the confined phase. Finally Section 5 is devoted to the conclusions.

2. A new model of holographic QCD by D4-branes and D0-branes

In this section, we propose a new model of holographic QCD which consists of N_4 color D4-branes, N_0 D0-branes and N_f flavor D8 and anti-D8-branes. The D0-branes are smeared on the color D4-branes and induce a gluon condensate. The theory has $U(N_4)$ color gauge symmetry and $U(N_f) \times U(N_f)$ chiral symmetry. The flavor D-branes are treated as a probe embedded into the background produced by the color D-branes.

2.1. D4+D0 solution in IIA supergravity

We consider N_4 D4-branes on which N_0 (Euclidean) D0-branes are homogeneously smeared. The configuration of these D-branes are shown in Table 1.

	X^0	X^1	X^2	X^3	X^4	ρ	6	7	8	9
N_4 D4-branes	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					
N_0 D0-branes	$\sim$	$\sim$	$\sim$	$\sim$	$\circ$					

Table 1 The configuration of the D4-branes and D0-branes. "$\sim$" stands for a smeared direction.

In IIA supergravity the non-extremal black brane solution describing such D-branes has already been known [26–30]. The solution has the following metric and dilaton:

$$ds^2 = \sqrt{\frac{H_0}{H_4}} dX_\mu dX^\mu + \frac{F}{\sqrt{H_0 H_4}} \left(dX^4\right)^2 + \sqrt{H_0 H_4} \left(\frac{d\rho^2}{F} + \rho^2 d\Omega_4^2\right), \tag{2.1a}$$

$$e^\phi = g_s \left(\frac{H_0^3}{H_4}\right)^{\frac{1}{4}}, \quad g_s := e^{-\phi_0}, \tag{2.1b}$$

where $\mu = 0, 1, 2, 3$ which are contracted by $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. H_0, H_4 and F are the harmonic functions:

$$H_0(\rho) = 1 + \left(\frac{\rho_0}{\rho}\right)^3, \quad H_4(\rho) = 1 + \left(\frac{\rho_4}{\rho}\right)^3, \quad F(\rho) = 1 - \left(\frac{\rho_h}{\rho}\right)^3. \tag{2.2}$$

ρ_0 and ρ_4 are related with the horizon, ρ_h, and the D-brane charges, N_0 and N_4, by

$$\rho_0^3 = -\frac{1}{2}\rho_h^3 + \sqrt{\frac{1}{4}\rho_h^6 + \pi^2 \ell_s^6 g_s^2 \left(\frac{(2\pi\ell_s)^4 N_0}{V_4}\right)^2}, \quad \rho_4^3 = -\frac{1}{2}\rho_h^3 + \sqrt{\frac{1}{4}\rho_h^6 + \pi^2 \ell_s^6 g_s^2 N_4^2}, \tag{2.3}$$

where V_4 denotes the volume of the space on which the D0-branes are smeared, *i.e.*, $V_4 := \int dX^0 dX^1 dX^2 dX^3$.

There are two Ramond-Ramond (RR) fluxes, $G_{(4)}$ and $G_{(8)}$, which are associated with the D4-branes and D0-branes respectively. Since the fluxes obey $(2\pi\ell_s)^{-3} \int G_{(4)} = N_4$ and $(2\pi\ell_s)^{-7} \int G_{(8)} = N_0$ by the definition of the D-brane charges, one can write down the fluxes as

$$G_{(4)} = \frac{(2\pi\ell_s)^3 N_4}{\Omega_4} d\Omega_4, \quad G_{(8)} = \frac{(2\pi\ell_s)^7 N_0}{V_4 \Omega_4} dX^0 \wedge dX^1 \wedge dX^2 \wedge dX^3 \wedge d\Omega_4, \tag{2.4}$$

where Ω_4 is the volume of unit four-sphere, *i.e.*, $\Omega_4 = 8\pi^2/3$.

2.2. The field theory limit

Let us consider the Maldacena limit [31],

$$\frac{\rho}{\ell_s^2} =: U \text{ is fixed}, \quad \ell_s \to 0. \tag{2.5}$$

Since U has the dimension of energy, the system under the limit (2.5) lies in the finite energy configuration. We additionally fix the ratio,

$$\nu := \frac{(2\pi\ell_s)^4 N_0}{V_4} \frac{1}{N_4}, \tag{2.6}$$

to be finite. Since the leading terms of H_0 and H_4 as $\ell_s \to 0$ are[1]

$$H_0 = \frac{\pi g_s N_4}{U^3 \ell_s^3}\nu + \mathcal{O}(\ell_s^4)\,, \quad H_4 = \frac{\pi g_s N_4}{U^3 \ell_s^3} + \mathcal{O}(\ell_s^0)\,,$$

the metric (2.1a) and the dilaton (2.1b) are reduced to

$$\begin{aligned} ds^2 &= \sqrt{\nu} dX_\mu dX^\mu + \frac{\ell_s^3 U^3}{\pi g_s N_4 \sqrt{\nu}} F_U \left(dX^4\right)^2 + \frac{\pi g_s N_4 \sqrt{\nu}}{\ell_s^3 U^3}\left(\frac{dU^2}{F_U} + U^2 d\Omega_4^2\right), \\ e^\phi &= g_s \nu^{\frac{3}{4}} \sqrt{\frac{\pi g_s N_4}{\ell_s^3 U^3}}\,, \quad F_U = 1 - \left(\frac{U_h}{U}\right)^3, \end{aligned} \tag{2.7}$$

where $U_h := \rho_h \ell_s^{-2}$. Rescaling the radial coordinate back, we can rewrite (2.7) as

$$ds^2 = \sqrt{\nu} dX_\mu dX^\mu + \left(\frac{\rho}{R}\right)^3 \frac{F(\rho)}{\sqrt{\nu}} \left(dX^4\right)^2 + \sqrt{\nu}\left(\frac{R}{\rho}\right)^3 \left(\frac{d\rho^2}{F(\rho)} + \rho^2 d\Omega_4^2\right), \tag{2.8a}$$

$$e^\phi = g_s \nu^{\frac{3}{4}} \left(\frac{R}{\rho}\right)^{\frac{3}{2}}, \quad F(\rho) = 1 - \left(\frac{\rho_h}{\rho}\right)^3, \quad R^3 := \pi g_s \ell_s^3 N_4\,. \tag{2.8b}$$

Simultaneously the RR fluxes (2.4) are described as

$$G_{(4)} = \frac{(2\pi\ell_s)^3 N_4}{\Omega_4} d\Omega_4\,, \quad G_{(8)} = \frac{(2\pi\ell_s)^3 N_4 \nu}{\Omega_4} dX^0 \wedge dX^1 \wedge dX^2 \wedge dX^3 \wedge d\Omega_4\,. \tag{2.9}$$

We note that at the $\nu \to 0$ limit the metric (2.8) does not become one in the field theory limit of N_4 D4-branes (without a D0-brane). In order to obtain it, we have to set $N_0 = 0$ before taking the field theory limit. Therefore our D4+D0-branes background (2.8) is not valid at $\nu = 0$ and is essentially different from the D4-branes background.

For later convenience, we rescale the coordinates by R as

$$x^\mu := \frac{X^\mu}{R}\,, \quad y := \frac{X^4}{R}\,, \quad r := \frac{\rho}{R}\,,$$

---

[1] If ν is not fixed, the small ℓ_s expansions of H_0 and H_4 become

$$H_0 = 1 + \frac{\pi^2 g_s^2 \ell_s^2}{U_h^3 U^3}\left(\frac{(2\pi)^4 N_0}{V_4}\right)^2 + \mathcal{O}(\ell_s^4)\,, \quad H_4 = \frac{\pi g_s N_4}{U^3 \ell_s^3} + \mathcal{O}(\ell_s^0)\,.$$

To the leading order of ℓ_s, H_0 is approximately equal to one, namely, the D0-branes do not affect the geometry, and the resulting metric and dilaton are nothing but those generated by only D4-branes. The metric containing the subleading term in H_0 was studied by [32–34].

so that x^μ, y and r are dimensionless coordinates. Then the metric and dilaton (2.8) become

$$R^{-2}ds^2 = \sqrt{\nu}dx_\mu dx^\mu + \frac{r^3 f_M(r)}{\sqrt{\nu}}dy^2 + \frac{\sqrt{\nu}}{r^3}\left(\frac{dr^2}{f_M(r)} + r^2 d\Omega_4^2\right), \tag{2.10a}$$

$$e^\phi = g_s\frac{\nu^{3/4}}{r^{3/2}}, \quad f_M(r) = 1 - \left(\frac{r_M}{r}\right)^3, \quad R^3 = \pi g_s \ell_s^3 N_4, \tag{2.10b}$$

$$\nu = \frac{(2\pi\ell_s)^4 N_0}{R^4 v_4}\frac{1}{N_4}, \quad v_4 := \int d^4x = \frac{V_4}{R^4}, \tag{2.10c}$$

where $r_M := \rho_h/R$, while the RR fluxes (2.9) are

$$G_{(4)} = \frac{(2\pi\ell_s)^3 N_4}{\Omega_4}d\Omega_4, \quad G_{(8)} = \frac{(2\pi\ell_s)^3 R^4 N_4 \nu}{\Omega_4}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\Omega_4. \tag{2.11}$$

We compactify the y direction by a circle with a period β_y in the same manner as [24], so that we obtain the four-dimensional dual gauge theory. In order for the smoothness at $r = r_M$, the period β_y has to be

$$\beta_y = \frac{4\pi\sqrt{\nu}}{3r_M^2}.$$

Then the Kaluza-Klein mass and the Yang-Mills coupling in the four-dimensional dual gauge theory are described as

$$M_{\mathrm{KK}} = \frac{2\pi}{R\beta_y} = \frac{3r_M^2}{2\sqrt{\nu}R}, \quad g_{\mathrm{YM}}^2 = 2\pi g_s \ell_s M_{\mathrm{KK}}. \tag{2.12}$$

The background given by (2.10) and (2.11) corresponds to the confined phase of the dual gauge theory. If we take the T-dual on the y direction, this D4+D0 becomes the D3+D(-1) system which was studied by [25]. The D(-1)-brane plays a role of gluon condensation in the dual gauge theory, because a Dp-brane can be realized as an instanton on D($p+4$)-branes. Therefore the D0-branes smeared on the D4-branes behave as the instantons and provide the gluon condensation.

We need to clarify the parameter region in which the supergravity approximation is valid. First, in order to suppress the loop correction, the effective string coupling must be small, namely, $e^\phi \ll 1$. Since e^ϕ decreases as r, this constraint is realized as $g_s\nu^{3/4}r_M^{-3/2} \ll 1$, that is,

$$\sqrt{\nu}\lambda \ll N_4^{\frac{4}{3}}(\ell_s M_{\mathrm{KK}})^2, \tag{2.13}$$

where λ is the 't Hooft coupling, $\lambda := g_{\rm YM}^2 N_4$. Second, the curvature must be smaller than ℓ_s^{-2} for neglecting a higher curvature correction. We calculate the scalar curvature from the metric (2.10a),

$$\mathcal{R}(r) = \frac{9}{\sqrt{\nu} R^2 r^2} (r_M^3 + r^3) \,. \tag{2.14}$$

Since this is the increasing function for large r, it is necessary to introduce the UV cutoff[2] r_∞ so that $\mathcal{R}(r_\infty) \lll \ell_s^{-2}$, namely, $r_\infty \lll \sqrt{\nu} R^2 \ell_s^{-2}$. This constraint with $r_M < r_\infty$ leads to

$$(\ell_s M_{\rm KK})^2 \lll \sqrt{\nu} \lambda \,. \tag{2.15}$$

When we assume that $\ell_s M_{\rm KK}$ is finite, (2.13) and (2.15) are consistent with the 't Hooft limit, *i.e.*, the $N_4 \to \infty$ limit with the large fixed λ. Since we have taken the small ℓ_s limit, the finiteness of $\ell_s M_{\rm KK}$ implies that $M_{\rm KK}$ is large, in other words, the compactification period β_y is small. Therefore the dual gauge theory flows to be four-dimensional. We note that, in holographically computing physical values of the gauge theory, ℓ_s and $M_{\rm KK}$ always appear in the combination of $\ell_s M_{\rm KK}$.

We can also write down the background geometry corresponding to the deconfined phase. By exchanging the coordinates and double Wick-rotation, namely, $(x^0, y) \to (iy, ix^0)$, in (2.10) and (2.11), we obtain the metric,

$$R^{-2} ds^2 = -\frac{r^3 f_T(r)}{\sqrt{\nu}} (dx^0)^2 + \sqrt{\nu} (d\vec{x}^2 + dy^2) + \frac{\sqrt{\nu}}{r^3} \left(\frac{dr^2}{f_T(r)} + r^2 d\Omega_4^2 \right) , \tag{2.16a}$$

$$e^\phi = g_s \frac{\nu^{3/4}}{r^{3/2}} \,, \quad f_T(r) = 1 - \left(\frac{r_T}{r} \right)^3 , \quad R^3 = \pi g_s \ell_s^3 N_4 \,, \tag{2.16b}$$

$$\nu = \frac{(2\pi\ell_s)^4 N_0}{R^4 v_4} \frac{1}{N_4} \,, \quad v_4 := \int dx^1 dx^2 dx^3 dy \,, \tag{2.16c}$$

and the RR fluxes,

$$G_{(4)} = \frac{(2\pi\ell_s)^3 N_4}{\Omega_4} d\Omega_4 \,, \quad G_{(8)} = \frac{(2\pi\ell_s)^3 R^4 N_4 \nu}{\Omega_4} dx^1 \wedge dx^2 \wedge dx^3 \wedge dy \wedge d\Omega_4 \,. \tag{2.17}$$

[2] In our field theory limit, the harmonic function H_0 in (2.2) is reduced to $H_0 = \nu / r^3$. Let us naively imagine another limit where H_0 has the form, $H_0 = 1 + \hat{\nu}/\hat{r}^3$ (*cf.* [32,33]). The scalar curvature in this case is

$$\hat{\mathcal{R}}(\hat{r}) = \frac{9}{\sqrt{\hat{\nu}} R^2 \hat{r}^2 (1 + \hat{r}^3/\hat{\nu})^{5/2}} \left[\hat{r}_M^3 + \hat{r}^3 - \frac{\hat{r}^3}{\hat{\nu}} (\hat{r}^3 - 2\hat{r}_M^3) - \frac{\hat{r}^6}{4\hat{\nu}^2} (\hat{r}^3 - \hat{r}_M^3) \right] .$$

At small $\hat{r}$, this curvature coincides with (2.14). In other words, our theory focuses on the small $\hat{r}$ region in the theory of $H_0 = 1 + \hat{\nu}/\hat{r}^3$.

The transition between the background geometry for the confined phase (2.10) and the one for the deconfined phase (2.16) is known as the Hawking-Page transition. Since we exchanged the time direction with the radial one, the configuration of D-branes is slightly modified as Table 2.

	x^0	x^1	x^2	x^3	y	r	6	7	8	9
N_4 D4-branes	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					
N_0 D0-branes	$\circ$	$\sim$	$\sim$	$\sim$	$\sim$					

Table 2 The configuration of the D4-branes and D0-branes for the deconfined phase.

Since the geometry (2.16) is a black hole, we can introduce a temperature by compactifying the Euclidean time direction τ $(:= ix^0)$ with a period β_τ which is determined to be $(4\pi/3)\sqrt{\nu}r_T^{-2}$. Then it leads to the (Hawking) temperature,

$$T = \frac{1}{R\beta_\tau} = \frac{3r_T^2}{4\pi\sqrt{\nu}R}\,. \tag{2.18}$$

It is remarkable that the power of effective couplings e^ϕ, (2.10b) and (2.16b), has a negative value, $-3/2$. This implies the UV asymptotic freedom. On the other hand, the dilaton in the D4-branes background without D0-branes behaves like $e^\phi \sim r^{+3/4}$ [4–6], that is, asymptotically non-free. Therefore we expect that the UV behavior in our holographic QCD model is improved by virtue of the D0-branes.

3. Dense baryons in the confined phase

3.1. The action of flavor D8-branes

In order to consider the flavor dynamics in the confined phase, we embed N_f D8-branes and N_f anti-D8-branes into the background (2.10), in which the D8-branes and anti-D8-branes are connected with each other, so that they become N_f U-shape D8-branes (see fig. 1). This implies the chiral symmetry breaking from $U(N_f) \times U(N_f)$ to $U(N_f)$.

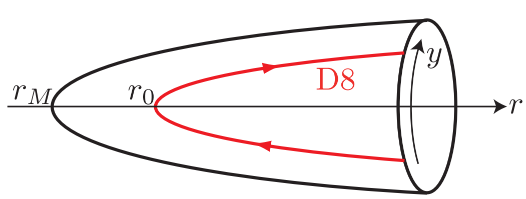

fig. 1 The embedding of D8-branes.

The D8-branes are stretched on $(x^0, x^1, x^2, x^3, r, S^4)$ and the collective coordinate y depends only on r (see Table 3).

	x^0	x^1	x^2	x^3	y	r	6	7	8	9
N_4 D4-branes	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					
N_0 D0-branes	$\sim$	$\sim$	$\sim$	$\sim$	$\circ$					
N_f D8-branes	$\circ$	$\circ$	$\circ$	$\circ$	$y(r)$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$

Table 3 The embedding of the flavor D8-branes.

Then the action of D8-branes without a worldvolume gauge field is

$$S_8 = -\mathcal{T} \int d^4x dr\, \nu \sqrt{r^2 f_M(r) \big(y'(r)\big)^2 + \frac{\nu}{r^4 f_M(r)}}\,, \tag{3.1}$$

where $\mathcal{T} = N_f \mu_8 R^9 \Omega_4 / g_s$, $\mu_8 = (2\pi)^{-8} \ell_s^{-9}$ and $'$ denotes the derivative with respect to r. We calculate the equation of motion from the action (3.1),

$$\frac{d}{dr}\left[r^2 f_M(r) y'(r) \left(r^2 f_M(r) \big(y'(r)\big)^2 + \frac{\nu}{r^4 f_M(r)} \right)^{-1/2} \right] = 0\,. \tag{3.2}$$

Integrating this equation with respect to r and imposing the boundary condition,

$$y(r_0) = 0\,, \quad y'(r_0) = \infty\,, \tag{3.3}$$

we obtain

$$\big(y'(r)\big)^2 = \frac{\nu}{r^6 \big(f_M(r)\big)^2} \left(\frac{r^2 f_M(r)}{r_0^2 f_M(r_0)} - 1 \right)^{-1}. \tag{3.4}$$

Therefore we can describe the separation between the D8-branes at the UV cutoff, $r = r_\infty$, as

$$l = 2 \int_{r_0}^{r_\infty} dr\, y'(r) = 2 \int_{r_0}^{r_\infty} dr\, \frac{\nu}{r^3 f_M(r)} \left(\frac{r^2 f_M(r)}{r_0^2 f_M(r_0)} - 1 \right)^{-\frac{1}{2}}. \tag{3.5}$$

Now let us turn on the baryons which are homogeneously distributed in $\mathbb{R}^3(x^1, x^2, x^3)$. These baryons are realized by baryon vertices, that is, the (baryonic) D4-branes wrapping S^4 [7]. We assume that the baryon vertices are located on the D8-branes at $r = r_c$. From the viewpoint of the worldvolume theory on the D8-branes, the number of baryons corresponds to $U(1)$ charge, because the Chern-Simons action includes the term, $A_0^{U(1)} \mathrm{tr}(F^{SU(N_f)})^2$.

More concretely we study the action of D8-branes. It consists of the DBI action and the Chern-Simons action,

$$S_8 = S_{\rm DBI} + S_{\rm CS}\,.$$

The $U(N_f)$ worldvolume gauge field, $\mathcal{A}$, on the D8-branes can be decomposed as

$$\mathcal{A} = A^{SU(N_f)} + \frac{1}{\sqrt{2N_f}} A^{U(1)}\,.$$

Since we are interested in the U(1) baryon charge, we focus on the time component of $U(1)$ gauge field, $A_0^{U(1)}$, which depends only on r, and we assume that the other components are equal to zero. For later convenience we define the dimensionless field by

$$a(r) := \frac{2\pi\ell_s^2}{\sqrt{2N_f}R^2} A_0^{U(1)}\,. \tag{3.6}$$

Due to this gauge field, the DBI action (3.1) is modified as

$$S_{\rm DBI} = -\mathcal{T}\int d^4x dr\, \mathcal{L}_{\rm DBI}\,, \quad \mathcal{L}_{\rm DBI} = \nu\sqrt{r^2 f_M(r)\big(y'(r)\big)^2 + \frac{\nu}{r^4 f_M(r)} - \frac{1}{r}\big(a'(r)\big)^2}\,. \tag{3.7}$$

We introduce the baryon vertices at $r = r_c$. We then have to take into account the Chern-Simons term coupling with the four-form flux $G_{(4)}$,

$$\frac{\mu_8(2\pi\ell_s^2)^3}{3!}\int G_{(4)}\wedge\omega_{(5)} = \frac{N_4}{24\pi^2}\int\omega_{(5)}\,, \quad \omega_{(5)} = {\rm Tr}\Big(\mathcal{A}\mathcal{F}^2 - \frac{1}{2}\mathcal{A}^3\mathcal{F} + \frac{1}{10}\mathcal{A}^5\Big)\,, \tag{3.8}$$

because this includes the term inducing the baryon charge,

$$S_{\rm CS} = \frac{N_4 N_f}{24\pi^2}\int d^4x dr \frac{3}{\sqrt{2N_f}} A_0^{U(1)} {\rm tr}\big(F^{SU(N_f)}\big)^2\,. \tag{3.9}$$

Since the baryon vertices are at $r = r_c$ and are homogeneously distributed in $\mathbb{R}^3$, ${\rm tr}F^2$ is identified with the baryon number density as

$$\frac{1}{8\pi^2}{\rm tr}\big(F^{SU(N_f)}\big)^2 = n_B\delta(r - r_c)\,. \tag{3.10}$$

Therefore (3.9) becomes

$$S_{\rm CS} = \mathcal{T}\int d^4x dr\, 3N_B a(r)\delta(r - r_c)\,, \quad N_B := n_B\Big(\frac{2\pi\ell_s}{R}\Big)^4\,. \tag{3.11}$$

We note that, although there is the other RR flux, $G_{(8)}$, in (2.11), it does not couple with $a(r)$, the time component of $U(1)$ gauge field.[3]

[3] Compared with the source of baryon charge (3.8), $(2\pi\ell_s^2)^3 G_{(4)}\wedge A_0^{U(1)}\wedge\mathcal{F}^2 \sim \mathcal{O}(\ell_s^9)$, we can neglect $(2\pi\ell_s^2)^4 * G_{(8)}\wedge A_0^{U(1)}\wedge\mathcal{F}^3 \sim \mathcal{O}(\ell_s^{11})$ due to small ℓ_s. (See also (2.11).)

3.2. Force balance condition between D8-branes and baryonic D4-branes

Since the baryonic D4-branes are attached on the D8-branes at $r = r_c$, the force from D4-branes has to balance with that from the D8-branes [8].

We remind the total action of D8-branes given by (3.7) and (3.11), *i.e.*,

$$S_8 = \mathcal{T} \int d^4x dr \left[-\mathcal{L}_{\rm DBI} + 3N_B a(r)\delta(r - r_c) \right] , \tag{3.12a}$$

$$\mathcal{L}_{\rm DBI} = \nu \sqrt{r^2 f_M(r) \big(y'(r)\big)^2 + \frac{\nu}{r^4 f_M(r)} - \frac{1}{r}\big(a'(r)\big)^2} \, . \tag{3.12b}$$

The equations of motion for $y(r)$ and $a(r)$ are respectively

$$\frac{d}{dr}\left[\frac{r^2 f_M(r) y'(r)}{\mathcal{L}_{\rm DBI}} \right] = 0 \, , \tag{3.13a}$$

$$d'(r) = 3N_B \delta(r - r_c) \, , \tag{3.13b}$$

where we introduced the electric displacement field,

$$d(r) := -\frac{\delta \mathcal{L}_{\rm DBI}}{\delta a'(r)} = \frac{\nu^2 a'(r)}{r \mathcal{L}_{\rm DBI}} \, . \tag{3.14}$$

From (3.13b) we obtain

$$d(r) = 3N_B \, ,$$

that is, the electric displacement field becomes the constant proportional to the baryon density. With the boundary condition, $y'(r_0) = \infty$, we can solve (3.13a) as

$$\big(y'(r)\big)^2 = \frac{\nu}{r^6 \big(f_M(r)\big)^2} \left(\frac{r^2 f_M(r)(\nu^2 + r d^2)}{r_0^2 f_M(r_0)(\nu^2 + r_0 d^2)} - 1 \right)^{-1} . \tag{3.15}$$

We are now considering the baryonic D4-branes which are attached on the D8-branes at $r = r_c$. Then we can imagine two cases about the shape of D8-branes. These depend on the direction of force from the baryonic D4-branes.

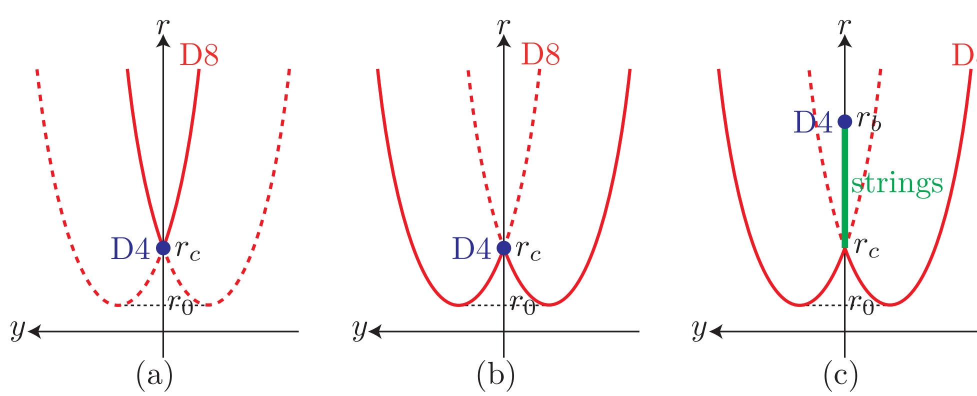

fig. 2 The force balance between the D8-branes and baryonic D4-branes.
(a) The V-shape D8-branes. (b,c) The W-shape D8-branes.

One is the case as fig. 2a where the V-shape D8-branes are pulled down by the falling D4-branes. We remember that this is the case for the generalized Sakai-Sugimoto model, *i.e.*, the D4-branes background without the D0-branes [8]. The other is as fig. 2b where the W-shape D8-branes are hung up by the floating D4-branes.

In order to clarify the shape of D8-branes in our D4+D0 model, we calculate the on-shell action of baryonic D4-branes. The baryon vertices with density n_B are identified with the $n_B v_3$ D4-branes wrapping S^4. Note that v_3 ($:= \int d^3x$) is the volume of $\mathbb{R}^3$. Therefore the on-shell action of baryonic D4-branes is

$$S_4 = -(n_B v_3)\mu_4 \int dx^0 d\Omega_4\, e^{-\phi}\sqrt{-\det g_{\rm D4}}\bigg|_{\text{on-shell}} =: -\int dx^0\, \mathcal{E}_4\,, \quad \mathcal{E}_4 = \frac{1}{3}\mathcal{T} v_3 d\sqrt{\frac{\nu}{r_c}}\,. \tag{3.16}$$

Since the energy, $\mathcal{E}_4$, decreases as r_c, the D4-branes yield an upward force (to the positive direction of r). Therefore our D4+D0 model prefers the W-shape D8-brane than the V-shape one. We also need to take into account the possibility as fig. 2c, in which the W-shape D8-branes and the baryonic D4-branes are connected by fundamental strings and the position of D4-branes is $r_b\,(\geq r_c)$ [35]. The action of the baryonic D4-branes and fundamental strings are described as

$$\begin{aligned} S_{4+\text{st}} &= -(n_B v_3)\left[\mu_4 \int dx^0 d\Omega_4\, e^{-\phi}\sqrt{-\det g_{\rm D4}} + \frac{1}{2\pi\ell_s^2}\int dx^0 \int_{r_c}^{r_b} dr\sqrt{-\det g_{\text{string}}}\right]_{\text{on-shell}} \\ &= -\mathcal{T} v_3 d\sqrt{\nu}\int dx^0\left[\frac{1}{3\sqrt{r_b}} + \int_{r_c}^{r_b}\frac{dr}{\sqrt{r^3 - r_M^3}}\right] =: -\int dx^0\, \mathcal{E}_{4+\text{st}}\,. \end{aligned}$$

Since the derivative of the energy $\mathcal{E}_{4+\text{st}}$ with respect to r_b is always positive, $\mathcal{E}_{4+\text{st}}$ has a minimum at $r_b = r_c$. In other words, the configuration like fig. 2b is favorable than that like fig. 2c. Finally the W-shape D8-branes on which the baryonic D4-branes directly attached (fig. 2c) is the (at least classically) stable configuration .

For the W-shape D8-branes we calculate the separation at the UV cutoff $r = r_\infty$ in terms of (3.15),

$$\begin{aligned} l &= 2\left(-\int_{r_c}^{r_0} + \int_{r_0}^{r_\infty}\right) dr\, |y'(r)| \\ &= 2\left(\int_{r_0}^{r_c} + \int_{r_0}^{r_\infty}\right) dr \frac{\sqrt{\nu}}{r^3 f_M(r)}\left(\frac{r^2 f_M(r)(\nu^2 + r d^2)}{r_0^2 f_M(r_0)(\nu^2 + r_0 d^2)} - 1\right)^{-\frac{1}{2}}. \end{aligned} \tag{3.17}$$

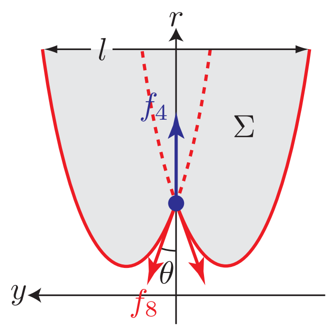

fig. 3 The force balance condition: $f_8 \cos\theta = f_4$.

Now let us calculate the force balance condition between the D8-branes and the baryonic D4-branes (see fig. 3).[4] The Legendre transform of the DBI action of D8-branes (3.7) is

$$\begin{aligned}\tilde{S}_8 &:= \mathcal{T} \int d^4x dr \Big(a'(r) d + \mathcal{L}_{\rm DBI} \Big) \\ &= \mathcal{T} \int d^4x dr \sqrt{(\nu^2 + r d^2) \left(r^2 f_M(r) \big(y'(r) \big)^2 + \frac{1}{r^4 f_M(r)} \right)} \,. \end{aligned} \tag{3.18}$$

From this equation, we can read that the effective tension at $r = r_c$ is

$$f_8 = \frac{\mathcal{T} v_3}{R} \nu^{\frac{1}{4}} \sqrt{\frac{\nu^2}{r_c} + d^2} \,. \tag{3.19}$$

The angle between the D8-branes and the r axis at $r = r_c$ is computed as

$$\cos\theta = \left. \frac{\sqrt{g_{rr}} dr}{\sqrt{g_{rr} dr^2 + g_{yy} dy^2}} \right|_{r=r_c} = \sqrt{1 - \frac{r_0^2 f_M(r_0)(\nu^2 + r_0 d^2)}{r_c^2 f_M(r_c)(\nu^2 + r_c d^2)}} \,. \tag{3.20}$$

The D8-branes provide the force, $f_8 \cos\theta$, along the r direction. On the other hand, the force from the D4-branes is evaluated in terms of (3.16),

$$f_4 = - \frac{1}{\sqrt{g_{rr}}} \left. \frac{d\mathcal{E}_4}{dr_c} \right|_{r=r_c} = \frac{\mathcal{T} v_3}{6R} \nu^{\frac{1}{4}} d \sqrt{f_M(r_c)} \,. \tag{3.21}$$

As a result, from (3.19), (3.20) and (3.21), we obtain the force balance condition, $f_8 \cos\theta = f_4$, namely,

$$\frac{1}{6} d = \sqrt{\frac{\nu^2 + r_c d^2}{r_c f_M(r_c)} \left(1 - \frac{r_0^2 f_M(r_0)(\nu^2 + r_0 d^2)}{r_c^2 f_M(r_c)(\nu^2 + r_c d^2)} \right)} \,. \tag{3.22}$$

[4] In this paper we calculate the forces in the left half ($y(r) \geq 0$). Therefore the actual total forces are twice and balance as $2 f_8 \cos\theta = 2 f_4$.

3.3. Chiral condensate

In order to consider the chiral condensate, we adopt the method suggested by [19], in which the open Wilson line operator,

$$\mathcal{O}_i^j(x^\mu) = \psi_L^{\dagger j}(x^\mu, y = -l/2)\mathcal{P}e^{\int_{-l/2}^{l/2} dy(iA_y+\Phi)}\psi_{Ri}(x^\mu, y = l/2)\,,$$

was introduced. i, j are indices of the fundamental representation of $U(N_f)$. The AdS/CFT correspondence allows us to evaluate the vacuum expectation value of the open Wilson line operator by the use of the on-shell action of fundamental string whose boundary is the flavor D8-branes (see fig. 3), that is to say,

$$\langle \mathcal{O}_i^j \rangle \simeq \delta_i^j \langle \mathcal{O} \rangle\,, \quad \langle \mathcal{O} \rangle = e^{-S_{\mathcal{O}}}\,, \quad S_{\mathcal{O}} = \frac{1}{2\pi\ell_s^2}\int_\Sigma d^2\sigma\sqrt{\det g}\,,$$

to leading order of α' $(= \ell_s^2)$. One can realize $\langle \mathcal{O} \rangle$ as the chiral condensate $\langle \bar{\psi}\psi \rangle$. $S_{\mathcal{O}}$ is, in other words, the area of the worldsheet instanton. By the use of (3.17), $S_{\mathcal{O}}$ can be written down as

$$S_{\mathcal{O}} = \frac{R^2}{\pi\ell_s^2}\int_0^{l/2} dy \int_{r(y)}^{r_\infty} dr = \frac{R^2}{\pi\ell_s^2}\left(\int_{r_0}^{r_c} + \int_{r_0}^{r_\infty}\right) dr(r_\infty - r)|y'(r)|\,. \tag{3.23}$$

We note that $S_{\mathcal{O}}$ does not have UV divergence, because we have introduced the UV cutoff, r_∞, in our model as we have discussed in Section 2. However $S_{\mathcal{O}}$ still has a large value depending on the cutoff. So we consider $\langle \mathcal{O} \rangle$ divided by the large constant,

$$\langle \mathcal{O} \rangle_\infty := \exp\left[-\frac{R^2}{2\pi\ell_s^2} l(r_\infty - r_M)\right]. \tag{3.24}$$

Let us evaluate the chiral condensate numerically. In order to sweep out r_M from numerical computations, we rescale the variables as

$$w := \frac{r}{r_M}\,, \quad \hat{d} := \sqrt{r_M}d\,, \quad \hat{l} := r_M^2 l\,.$$

Then the force balance condition (3.22) is rewritten as

$$w_0^2(1 - w_0^{-3})(\nu^2 + w_0\hat{d}^2) = w_c^2(1 - w_c^{-3})\left(\nu^2 + w_c\hat{d}^2\frac{35 + w_c^{-3}}{36}\right) =: Q(\nu, \hat{d}, w_c)\,.$$

This equation determines w_0 as a function of ν, $\hat{d}$ and w_c. From (3.17), (3.23) and (3.24), the separation and chiral condensate are described as

$$\hat{l} = 2\left(\int_{w_0}^{w_c} + \int_{w_0}^{w_\infty}\right) dw \frac{\sqrt{\nu}}{w^3 - 1}\left(\frac{w^2(1-w^{-3})(\nu^2 + w\hat{d}^2)}{Q(\nu, \hat{d}, w_c)} - 1\right)^{-\frac{1}{2}}, \tag{3.25}$$

$$\frac{\langle \mathcal{O} \rangle}{\langle \mathcal{O} \rangle_\infty} = \exp\left(\frac{R^2}{\pi \ell_s^2 r_M} I\right),$$

$$I = \left(\int_{w_0}^{w_c} + \int_{w_0}^{w_\infty}\right) dw \frac{\sqrt{\nu}}{w^2 + w + 1}\left(\frac{w^2(1-w^{-3})(\nu^2 + w\hat{d}^2)}{Q(\nu, \hat{d}, w_c)} - 1\right)^{-\frac{1}{2}}. \tag{3.26}$$

By fixing ν, $\hat{l}$ and w_∞, we calculate the density $\hat{d}$ dependence of $\exp(I)$.

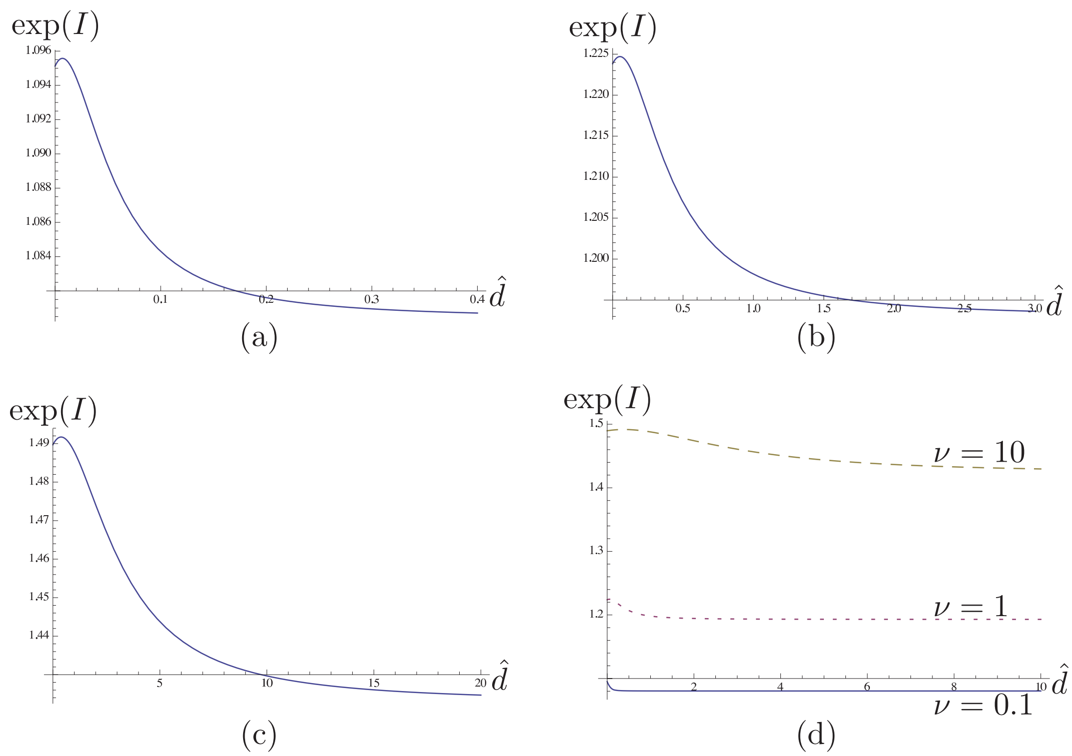

fig. 4 The plots of the density $\hat{d}$ dependence of $\exp(I)$ with $w_\infty = 10^4$ and $\hat{l} = 0.1$ fixed.
(a) $\nu = 0.1$. (b) $\nu = 1$. (c) $\nu = 10$.
(d) The plots for ν=0.1 (solid), 1 (dotted), 10 (dashed). Every $\exp(I)$ converges to a certain constant at $\hat{d} \to \infty$.

For instance, when we set $w_\infty = 10^4$, $\hat{l} = 0.1$ and $\nu = 0.1, 1, 10$, we obtain fig. 4. Note that the contribution from large w to the integrations in (3.25) and (3.26) is very small, therefore $\exp(I)$ is not sensitive to the cutoff w_∞. We can see in fig. 4 that the chiral condensate, $\langle \mathcal{O} \rangle$ $(= \langle \mathcal{O} \rangle_\infty (e^I)^{R^2/(\pi \ell_s^2 r_M)})$, increases in very low density. This is different

from our intuition from ordinary QCD. On the other hand, in high density, the chiral condensate is a monotonically decreasing function of $\hat{d}$ and converges to a finite value as $\hat{d} \to \infty$. In all other known models of holographic QCD [12–15], the chiral condensate increases in high density and it is completely opposite to the expectation from ordinary QCD. Therefore one can say that our model drastically improves the high density behavior of chiral condensate.

4. Comments on the deconfined phase

4.1. The action of flavor D8-branes

The background corresponding to the deconfined phase has been given by (2.16) and (2.17). We embed the N_f D8-branes (and N_f anti-D8-branes) into it in the same manner as we did in the confined phase, that is to say, we identify the worldvolume coordinates of the D8-branes with $(x^0, x^1, x^2, x^3, r, \Omega_4)$ and assume that the collective coordinate, y, depends only on r. Since we are interested in the $U(1)$ baryon charge, we take into account the $U(1)$ gauge field denoted by (3.6). Then the DBI action is described as

$$S_{\rm DBI}^{(\rm dec)} = -\mathcal{T} \int d^4x dr \, \mathcal{L}_{\rm DBI}^{(\rm dec)} \, , \quad \mathcal{L}_{\rm DBI}^{(\rm dec)} = \nu \sqrt{r^2 f_T(r) \big(y'(r)\big)^2 - \frac{1}{r} \big(a'(r)\big)^2 + \frac{1}{r}} \, . \qquad (4.1)$$

There are two Chern-Simons terms coupling with the $U(1)$ gauge field, $a(r)$. One is the coupling with the four-form flux (2.17), *i.e.*, $\int G_{(4)} \wedge \omega_{(5)}$, originated from the background D4-branes. As we explained in Section 3.1, $\omega_{(5)}$ includes $A^{U(1)}\mathrm{tr}(F^{SU(N_f)})^2$. Therefore it provides us the source term of the $U(1)$ baryon charge,

$$S_{\rm CS}^{(B)} = \mathcal{T} \int d^4x dr \, 3N_B a(r) \delta(r - r_c) \, , \qquad (4.2)$$

in the same way as (3.11). This also implies that the N_B baryonic D4-branes wrapping S^4 are located at $r = r_c$.

In the deconfined phase there is the other Chern-Simons term induced by the eight-form flux (2.17), $G_{(8)}$, of the D0-branes, namely, $\int G_{(8)} \wedge \mathcal{A}_0 dx^0$. This term includes

$$S_{\rm CS}^{(D0)} = \mathcal{T} \int d^4x dr \, 3\nu y'(r) a(r) \, . \qquad (4.3)$$

We note that this Chern-Simons term yields the $U(1)$ charge even if there is not a baryon ($N_B = 0$).

4.2. Without a baryon

In this subsection we concentrate on the case of no baryon, that is, $N_B = 0$. From (4.1) and (4.3), the action of D8-branes is

$$S_8^{(\mathrm{dec})}\big|_{N_B=0} = -\mathcal{T}\int d^4x dr \big[\mathcal{L}_{\mathrm{DBI}}^{(\mathrm{dec})} - 3\nu y'(r)a(r)\big]\,. \tag{4.4}$$

We calculate the equations of motion from this action,

$$\frac{d}{dr}\left[\frac{\nu r^2 f_T(r)y'(r)}{\mathcal{L}_{\mathrm{DBI}}^{(\mathrm{dec})}} - 3a(r)\right] = 0\,, \tag{4.5a}$$

$$\frac{d}{dr}\left[\frac{\nu a'(r)}{r\mathcal{L}_{\mathrm{DBI}}^{(\mathrm{dec})}} - 3y(r)\right] = 0\,. \tag{4.5b}$$

Since these equations are symmetric under the constant shifts of the fields, we redefined the fields by

$$\psi(r) := y(r) + C_1\,, \quad \alpha(r) := a(r) + C_2\,. \tag{4.6}$$

Then, integrating (4.5) with respect to r, we obtain

$$\psi'(r) = \frac{3\alpha(r)}{\sqrt{r^3 f_T(r)\Big\{9r^3 f_T(r)\big(\psi(r)\big)^2 - 9\big(\alpha(r)\big)^2 + r^2 f_T(r)\Big\}}}\,, \tag{4.7a}$$

$$\alpha'(r) = \frac{3\sqrt{r^3 f_T(r)}\psi(r)}{\sqrt{9r^3 f_T(r)\big(\psi(r)\big)^2 - 9\big(\alpha(r)\big)^2 + r^2 f_T(r)}}\,, \tag{4.7b}$$

where the integration constants are absorbed by C_1 and C_2. Note that, when $(\psi(r), \alpha(r))$ is a solution of (4.7), $(-\psi(r), -\alpha(r))$ is also a solution.

The trivial solution is $\alpha(r) = \psi(r) = 0$. When we put a pair of boundary conditions, $y(r_\infty) = \pm l/2$, this solution leads to a pair of solutions, $y(r) = \pm l/2$, which represents the parallel D8-branes and anti-D8-branes separated from each other by l. This configuration preserves the chiral symmetry, $U(N_f) \times U(N_f)$.

Let us look for a non-trivial solution. In order to find a U-shape solution whose tip is located at $r = r_0$, we impose the boundary condition:

$$\psi(r_0) = 0\,, \quad \psi'(r_0) = \infty\,. \tag{4.8}$$

From (4.7a), one can interpret $\psi'(r_0) = \infty$ to $r_0^2 f(r_0) - 9\big(\alpha(r_0)\big)^2 = 0$. Therefore the boundary condition (4.8) can be replaced with

$$\psi(r_0) = 0\,, \quad \alpha(r_0) = \frac{1}{3}r_0\sqrt{f(r_0)}\,. \tag{4.9}$$

We are now considering the $N_B = 0$ case, *i.e.*, there is no source induced by a baryon. Therefore there does not exit a cusp singularity on the D8-branes. In terms of the solution of (4.7) and (4.9), such a smooth U-shape D8-brane is described as

$$y(r) = \pm\psi(r)\,, \quad a(r) = \pm(\alpha(r) - \alpha(r_0))\,, \tag{4.10}$$

where C_1 and C_2 in (4.6) are determined so that $y(r)$ and $a(r(y))$ are smooth at $y(r_0) = 0$.

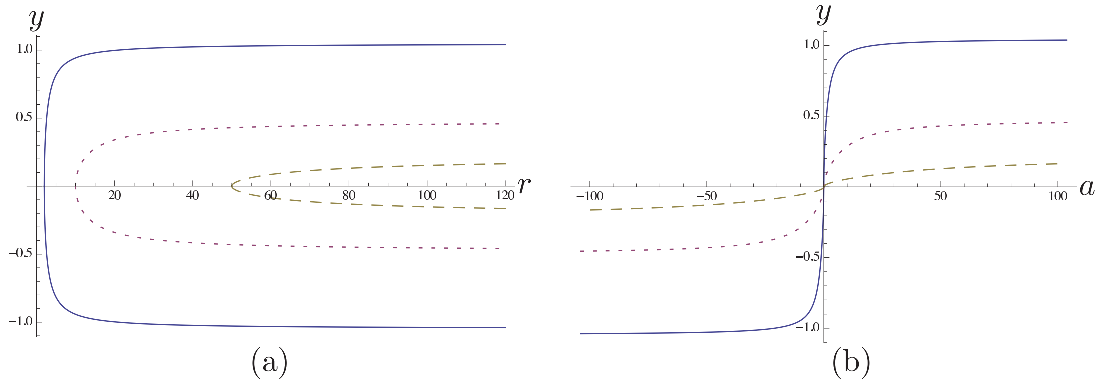

fig. 5 The plots of $y(r)$ and $a(r)$ with fixed $r_T = 1$ for r_0= 2 (solid), 10 (dotted) and 50 (dashed). (a) The plots of $y(r)$. (b) The parametric plots of $(a(r), y(r))$.

Since it is difficult to solve (4.7) analytically, we perform numerical computation. For instance, in fixing $r_T = 1$, we obtain the solutions depicted in fig. 5. The D8-branes and anti-D8-branes are smoothly connected at $r = r_0$, so that the U-shape D8-branes appear. This implies the chiral symmetry breaking from $U(N_f) \times U(N_f)$ to $U(N_f)$. Although the gauge field diverges at $r \to \infty$, we neglect the large r region. Because, as we discussed in Section 2.2, the UV cutoff r_∞ is necessary due to the validity of supergravity approximation.

4.3. With baryons

We shall turn on the baryons which are the $n_B v_3$ baryonic D4-branes located at $r = r_c$. The total action of D8-branes is given by (4.1), (4.2) and (4.3) because of $N_B \neq 0$. Although the equation of motion for $y(r)$ is same as (4.5a), the one for $a(r)$ is slightly different from (4.5b) due to the Chern-Simons term (4.2). By redefining

$$d(r) := -\frac{\delta\mathcal{L}_{\mathrm{DBI}}^{(\mathrm{dec})}}{\delta a'(r)} - 3\nu y(r)\,, \tag{4.11}$$

the equation of motion for $a(r)$ has a simple expression, $d'(r) = 3N_B\delta(r - r_c)$. Therefore $d(r)$ is a constant equal to $3N_B$.

In terms of (2.16) we write down the on-shell action of the baryonic D4-branes wrapping S^4 as

$$S_4^{(\rm dec)} = -\int dx^0 \mathcal{E}_4^{(\rm dec)}\,, \quad \mathcal{E}_4^{(\rm dec)} = \frac{1}{3}\mathcal{T} v_3 d r_c \sqrt{f_T(r_c)}\,. \tag{4.12}$$

Since the energy $\mathcal{E}_4^{(\rm dec)}$ monotonically increases as r_c, the baryonic D4-branes obtain a force toward the negative direction of r. Therefore this force from the D4-branes balances with the force from the V-shape D8-branes in a manner as fig. 2a. By using the solution of $\psi(r)$ and $\alpha(r)$ that we obtained in the previous subsection, we can describe the V-shape D8-brane as

$$y(r) = \pm(\psi(r) - \psi(r_c))\,, \quad a(r) = \pm(\alpha(r) - \alpha(r_0))\,. \tag{4.13}$$

Note that the gauge field $a(r)$ is not continuous at $r = r_c$ because there is the baryonic D4-branes, namely, the source of $U(1)$ charge.

The Legendre transformed action of D8-branes is

$$\tilde{S}_8^{(\rm dec)} = \mathcal{T}\int d^4x dr \sqrt{\left(\nu^2 + r(d + 3\nu y(r))^2\right)\left(r^2 f_T(r)\big(y'(r)\big)^2 + \frac{1}{r}\right)}\,, \tag{4.14}$$

up to total derivative. From (4.12) and (4.14) we calculate the force balance condition,

$$\frac{1}{3} d r_c \left[1 + \frac{1}{2}\left(\frac{r_T}{r_c}\right)^3\right] = \sqrt{\frac{f_T(r_c)(\nu^2 + r_c d^2)}{r_c^2 f_T(r_c)\big(y'(r_c)\big)^2 + r_c^{-1}}}\,. \tag{4.15}$$

The separation of V-shape D8-branes at $r = r_\infty$ is

$$l = 2\int_{r_c}^{r_\infty} |y'(r)| = 2(\psi(r_\infty) - \psi(r_c))\,. \tag{4.16}$$

In the same way as Section 3.3, we shall study the density dependence of the chiral condensate $\langle\mathcal{O}\rangle$ by

$$\begin{aligned}\frac{\langle\mathcal{O}\rangle}{\langle\mathcal{O}\rangle_\infty} &= \exp\left[\frac{R^2}{\pi\ell_s^2}\int_{r_c}^{r_\infty} dr \int_0^{y(r)} dy \frac{\sqrt{\nu}}{r^3 f_T(r)} - \frac{R^2}{\pi\ell_s^2}\int_{r_c}^{r_\infty} dr \int_0^{l/2} dy \frac{\sqrt{\nu}}{r^3 f_T(r)}\right] \\ &= \exp\left(\frac{R^2\sqrt{\nu}}{2\pi\ell_s^2} I_{(D)}\right), \quad I_{(D)} = \int_{r_c}^{r_\infty} dr \frac{2y(r) - l}{\sqrt{r^3 - r_T^3}}\,. \end{aligned} \tag{4.17}$$

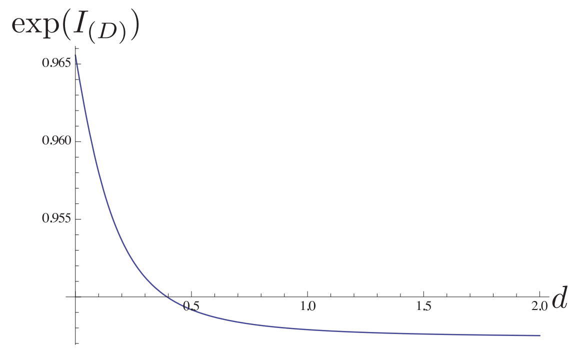

fig. 6 The plot of $\exp(I_{(D)})$ depending on the density d with fixed $l = 0.5$ and $r_T = \nu = 1$.

For instance we numerically evaluate $\exp(I_{(D)})$ with fixed $l = 0.5$ and $r_T = \nu = 1$, which is depicted in fig. 6. It shows that the chiral condensate monotonically decreases as the density d. This agrees very well with our expectation from the ordinary QCD. The $\exp(I_{(D)})$ for $l = 0.5$ and $r_T = \nu = 1$ is approximately equal to 0.966 at $d = 0$, and converges to a finite value, 0.947, at $d = \infty$ (see fig. 6).

5. Conclusions

In this paper we proposed a model of holographic QCD whose background has the asymptotic freedom. It is the specific field theory limit of the supergravity solution of the color D4-branes on which the D0-branes are smeared. In taking the limit, we fixed the ratio ν to be finite. Into this background we have introduced the flavor D8-branes and the baryonic D4-branes in order to study the chiral condensate in dense baryonic medium. The dilaton in the background of D4-branes without D0-branes behaves like $e^{\phi} \sim r^{3/4}$. On the other hand, the dilaton in our background behaves like $e^{\phi} \sim r^{-3/2}$, that is, the effective coupling is asymptotically free. Although it is still different from $(\log r)^{-1}$ in ordinary QCD, the D0-branes improve the asymptotic freedom in our model.

We have studied the chiral condensate by use of the open Wilson line, which we can evaluate by the area of open string worldsheet whose boundary is the D8-branes. In the confined phase the baryonic D4-branes balance with the D8-branes which have W-shape rather than V-shape. This is because the force from the D4-branes is toward the positive direction of the radial coordinate. This can be understood that the curvature increases for large radius so that the gravity is acting on baryon vertex upward.

Under this force balance condition we have calculated the chiral condensate concretely with the fixed $\hat{l}$, the separation between the D8-branes at the UV cutoff. The results for

three cases of ν are shown in fig. 4. All of the three have the common features as follows. In the region of very low density of baryons, the chiral condensate increases as the density. This behavior is not what one expects from the QCD. We attribute this anomaly to the approximation of the uniform distribution of baryons in the space $\mathbb{R}^3$, which is not a good one when the density of baryon is small. In high density we have obtained a remarkable result, that is, the chiral condensate monotonically decreases. This is in good agreement with our intuition from the ordinary QCD. When the density goes to infinity, the chiral condensate converges to a certain constant.

We have studied also the deconfined phase. We numerically solved the equations of motion without the baryons, and obtained the U-shape solution of D8-branes. It seems that the solution of the gauge field has UV divergence. However we do not need to seriously worry about the divergence, because basically the UV cutoff is necessary in our background due to the validity of supergravity approximation of bulk theory. Turning on baryons, we have calculated the chiral condensate. It is different from the confined phase that the D8-brane balancing with the baryonic D4-branes has V-shape in the deconfined phase. As a result, it monotonically decreases in whole range of the density (see fig. 6). This is in agreement with the expectation from ordinary QCD. In the deconfined phase there is the solution of parallel D8-branes, which preserves the chiral symmetry. Therefore we expect the chiral symmetry restoration occurs in certain high density [36]. One can guess that this transition is clarified by comparing the free energies of two configurations, namely, the V-shape D8-branes with baryon vertices and the parallel D8-branes. However there is subtlety due to the UV cutoff. Therefore we need to develop a way beyond the UV cutoff.

Acknowledgement

This work was supported by the NRF grant through the SRC program CQUeST with grant number 2005-0049409 and partially supported by the WCU project of Korean Ministry of Education, Science and Technology (R33-2008-000-10087-0).

References

[1] G. E. Brown and M. Rho, "Scaling effective Lagrangians in a dense medium," Phys. Rev. Lett. **66** (1991) 2720.

[2] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2** (1998) 231. [hep-th/9711200].

[3] A. Karch and E. Katz, "Adding flavor to AdS/CFT," JHEP **0206** (2002) 043. [hep-th/0205236].

[4] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, "Towards a holographic dual of large N_c QCD," JHEP **0405** (2004) 041. [hep-th/0311270].

[5] T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD," Prog. Theor. Phys. **113** (2005) 843. [hep-th/0412141].

[6] T. Sakai and S. Sugimoto, "More on a holographic dual of QCD," Prog. Theor. Phys. **114** (2005) 1083. [hep-th/0507073].

[7] E. Witten, "Baryons and branes in anti-de Sitter space," JHEP **9807** (1998) 006. [hep-th/9805112].

[8] O. Bergman, G. Lifschytz and M. Lippert, "Holographic nuclear physics," JHEP **0711** (2007) 056. [arXiv:0708.0326 [hep-th]].

[9] K. -Y. Kim, S. -J. Sin and I. Zahed, "Dense hadronic matter in holographic QCD," [hep-th/0608046].

[10] K. -Y. Kim, S. -J. Sin and I. Zahed, "The chiral model of Sakai-Sugimoto at finite baryon density," JHEP **0801** (2008) 002. [arXiv:0708.1469 [hep-th]].

[11] K. -Y. Kim, S. -J. Sin and I. Zahed, "Dense holographic QCD in the Wigner-Seitz approximation," JHEP **0809** (2008) 001. [arXiv:0712.1582 [hep-th]].

[12] S. Nakamura, Y. Seo, S. -J. Sin and K. P. Yogendran, "A new phase at finite quark density from AdS/CFT," J. Korean Phys. Soc. **52** (2008) 1734. [hep-th/0611021].

[13] Y. Seo and S. -J. Sin, "Baryon mass in medium with holographic QCD," JHEP **0804** (2008) 010. [arXiv:0802.0568 [hep-th]].

[14] M. Edalati and J. F. Vazquez-Poritz, "Chiral condensates in finite density holographic NJL model from string worldsheets," [arXiv:0906.5336 [hep-th]].

[15] S. Seki and S. -J. Sin, "Chiral condensate in holographic QCD with baryon density," JHEP **1208** (2012) 009. [arXiv:1206.5897 [hep-th]].

[16] R. Casero, E. Kiritsis and A. Paredes, "Chiral symmetry breaking as open string tachyon condensation," Nucl. Phys. B **787** (2007) 98. [hep-th/0702155 [hep-th]].

[17] K. Hashimoto, T. Hirayama and A. Miwa, "Holographic QCD and pion mass," JHEP **0706** (2007) 020. [hep-th/0703024 [hep-th]].

[18] O. Bergman, S. Seki and J. Sonnenschein, "Quark mass and condensate in HQCD," JHEP **0712** (2007) 037. [arXiv:0708.2839 [hep-th]].

[19] O. Aharony and D. Kutasov, “Holographic duals of long open strings,” Phys. Rev. D **78** (2008) 026005. [arXiv:0803.3547 [hep-th]].

[20] K. Hashimoto, T. Hirayama, F. -L. Lin and H. -U. Yee, “Quark mass deformation of holographic massless QCD,” JHEP **0807** (2008) 089. [arXiv:0803.4192 [hep-th]].

[21] A. Dhar and P. Nag, “Tachyon condensation and quark mass in modified Sakai-Sugimoto model,” Phys. Rev. D **78** (2008) 066021. [arXiv:0804.4807 [hep-th]].

[22] S. Seki, “Intersecting D4-branes model of holographic QCD and tachyon condensation,” JHEP **1007** (2010) 091. [arXiv:1003.2971 [hep-th]].

[23] J. M. Maldacena, “Wilson loops in large N field theories,” Phys. Rev. Lett. **80** (1998) 4859. [hep-th/9803002].

[24] E. Witten, “Anti-de Sitter space, thermal phase transition, and confinement in gauge theories,” Adv. Theor. Math. Phys. **2** (1998) 505. [hep-th/9803131].

[25] H. Liu and A. A. Tseytlin, “D3-brane–D-instanton configuration and $\mathcal{N} = 4$ superYM theory in constant selfdual background,” Nucl. Phys. B **553** (1999) 231. [hep-th/9903091].

[26] M. Cvetic and A. A. Tseytlin, “Nonextreme black holes from nonextreme intersecting M-branes,” Nucl. Phys. B **478** (1996) 181. [hep-th/9606033].

[27] M. S. Costa, “Composite M-branes,” Nucl. Phys. B **490** (1997) 202. [hep-th/9609181].

[28] I. Y. Aref’eva, K. S. Viswanathan and I. V. Volovich, “p-brane solutions in diverse dimensions,” Phys. Rev. D **55** (1997) 4748. [hep-th/9609225].

[29] M. S. Costa, “Black composite M-branes,” Nucl. Phys. B **495** (1997) 195. [hep-th/9610138].

[30] I. Aref’eva and A. Volovich, “Composite p-branes in diverse dimensions,” Class. Quant. Grav. **14** (1997) 2991. [hep-th/9611026].

[31] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, “Supergravity and the large N limit of theories with sixteen supercharges,” Phys. Rev. D **58** (1998) 046004. [hep-th/9802042].

[32] J. L. F. Barbon and A. Pasquinucci, “Aspects of instanton dynamics in AdS/CFT duality,” Phys. Lett. B **458** (1999) 288. [hep-th/9904190].

[33] K. Suzuki, “D0-D4 system and QCD_{3+1},” Phys. Rev. D **63** (2001) 084011. [hep-th/0001057].

[34] C. Wu, Z. Xiao and D. Zhou, “Sakai-Sugimoto model in D0-D4 background,” [arXiv:1304.2111 [hep-th]].

[35] S. Seki and J. Sonnenschein, “Comments on baryons in holographic QCD,” JHEP **0901** (2009) 053. [arXiv:0810.1633 [hep-th]].

[36] O. Aharony, J. Sonnenschein and S. Yankielowicz, “A holographic model of deconfinement and chiral symmetry restoration,” Annals Phys. **322** (2007) 1420. [hep-th/0604161].